RETURN PERIODS IN JULIA

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CEVE 543

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Waiting and risk

Return periods from data

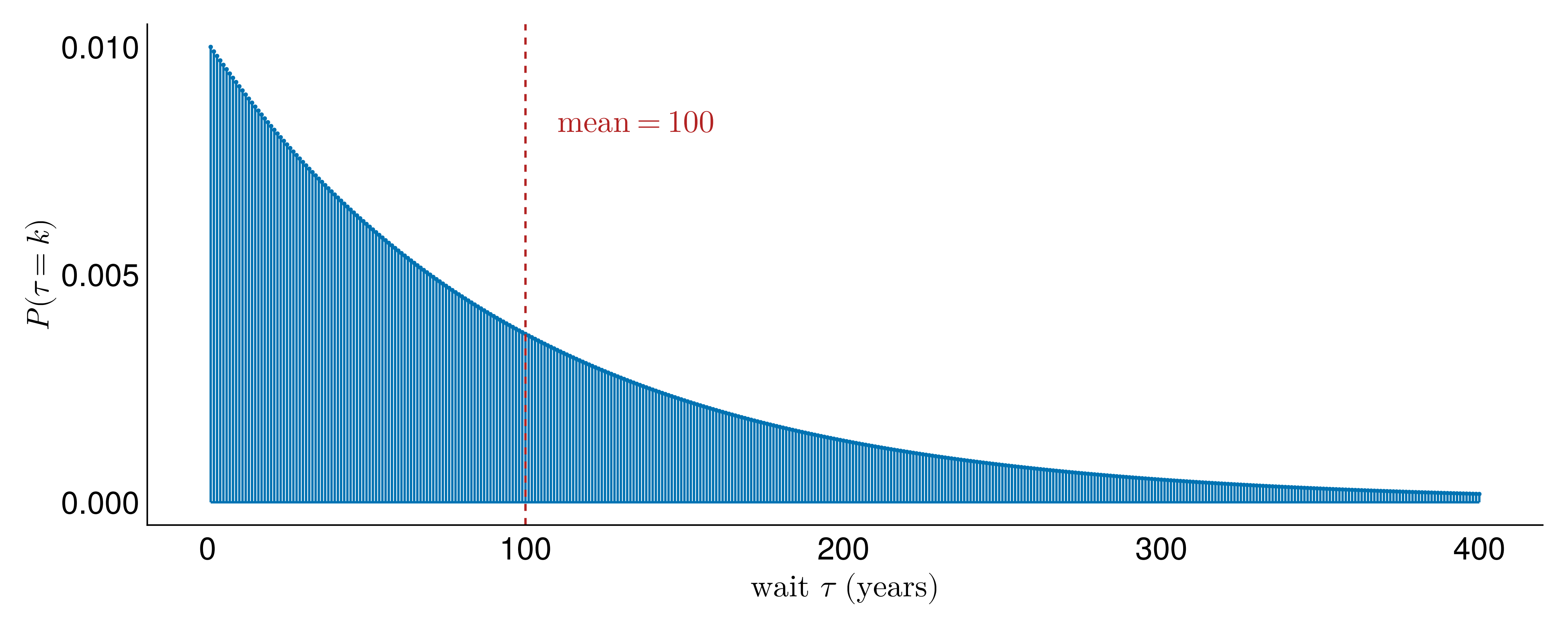
WAITING TIME FOR THE NEXT EXCEEDANCE, BY HAND

τ is the number of years until the "coin" next comes up "heads". By the geometric distribution:

$$p_\tau(k) = (1-p)^{k-1}p, \qquad E[\tau] = \frac{1}{p} = T$$

WAITING TIME FOR THE NEXT EXCEEDANCE, BY SIMULATION

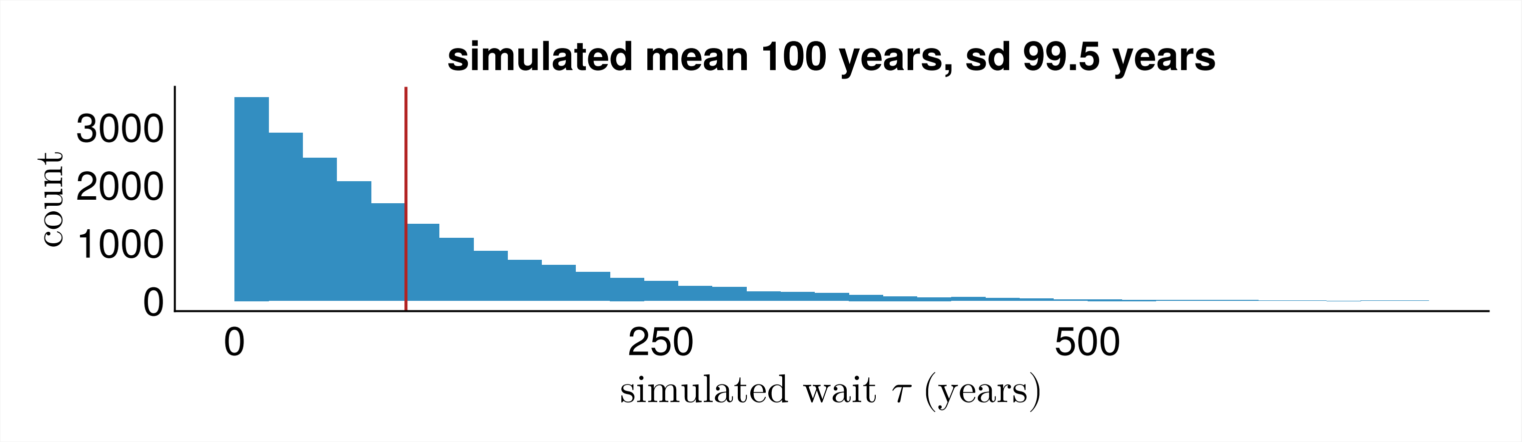
$p = 0.01$



HOW VARIABLE IS THAT WAIT?

$$\mathrm{Var}(\tau) = \frac{1-p}{p^2}, \qquad \mathrm{sd}(\tau) = \frac{\sqrt{1-p}}{p} \approx T \ \text{ for small } \ p$$

The spread is as large as the mean wait, so the mass near 1 and the long tail are the same picture.



HOW LIKELY ARE TWO EXCEEDANCES IN TEN YEARS, AT $p = 1/4$?

Ten flips of the bent coin, so $N = 10$, with Y the number of heads.

$$p_Y(y) = \binom{N}{y} p^y (1-p)^{N-y}$$

```julia
Y = Binomial(10, 0.25)
p_two_or_more = 1 - pdf(Y, 0) - pdf(Y, 1)
@printf("P(Y >= 2) = %.3g", p_two_or_more)
```

```
P(Y >= 2) = 0.756
```

A “4-year flood” turns up at least twice per decade more often than not.

RARER EVENTS: $p = 1/10$

Same ten flips, same count Y, a coin bent one tenth of the way instead of one quarter.

```
Y = Binomial(10, 0.1)
p_two_or_more = 1 - pdf(Y, 0) - pdf(Y, 1)
@printf("P(Y >= 2) = %.3g", p_two_or_more)
```

```
P(Y >= 2) = 0.264
```

We get two “ten-year floods” in a decade a quarter of the time.

WHAT RISK DOES A DESIGN LIFE CARRY?

$$R = 1 - (1 - p)^N, \qquad T = \frac{1}{1 - (1 - R)^{1/N}}$$

```julia
risk(T, N) = 1 - (1 - 1 / T)^N
T_needed(R, N) = 1 / (1 - (1 - R)^(1 / N))

@printf("100-year flood, 50-year life: R = %.3g\n", risk(100, 50))
@printf("10%% risk over a 5-year job needs T = %.3g years", T_needed(0.10, 5))
```

```
100-year flood, 50-year life: R = 0.395
10% risk over a 5-year job needs T = 48 years
```

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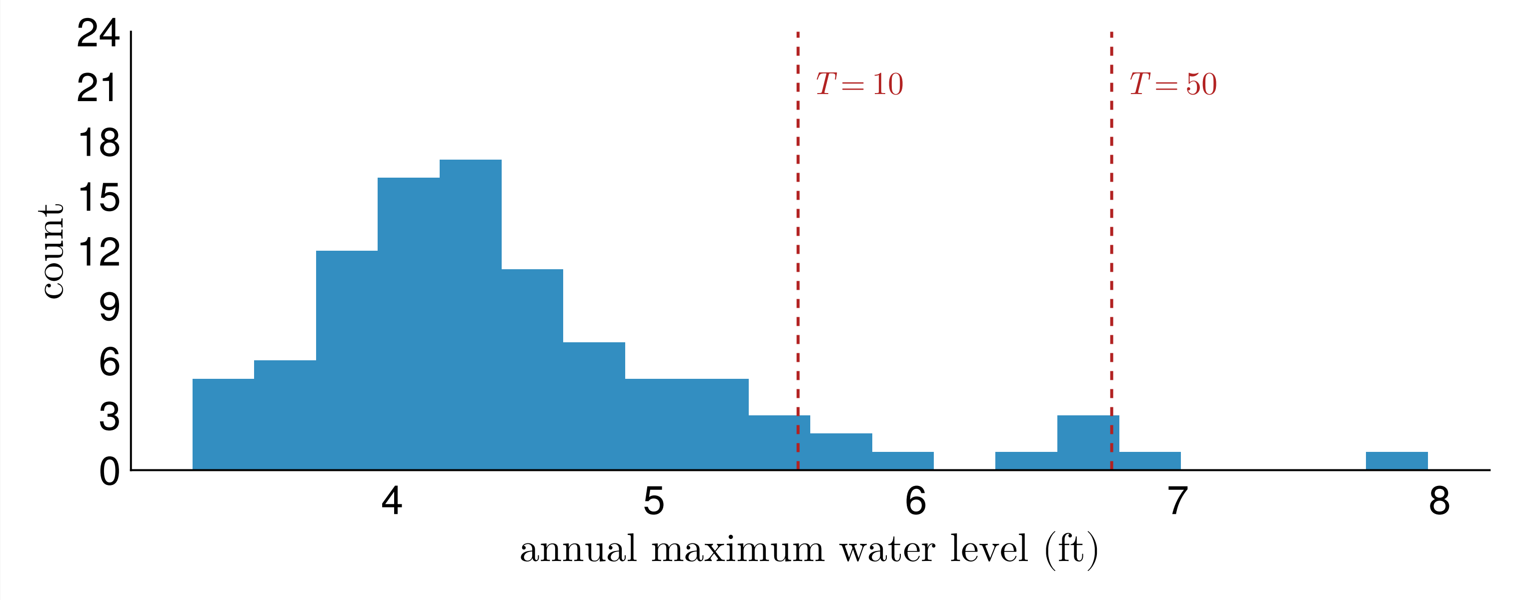
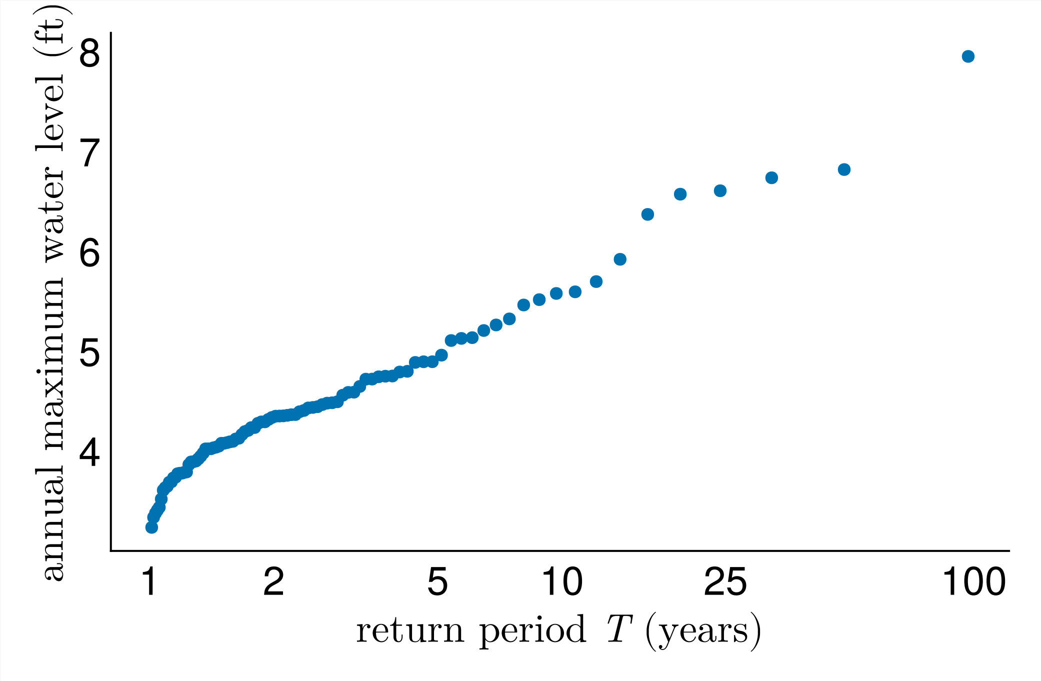
A HISTOGRAM HIDES THE TAIL

Histograms make it hard to compare and visualize tails.



WHAT DOES A RETURN PERIOD PLOT SHOW?



Which axis is data, and which is an estimate?

- Level: a number a gauge recorded.
- Return period: computed from the rank of that level in a finite record.

HOW IS THE RETURN PERIOD ESTIMATED?

Sort descending, so rank $i = 1$ is the largest, and a hat marks a quantity computed from the record rather than known.

$$\hat{p}_i = \frac{i}{n+1}, \qquad \hat{T}_i = \frac{1}{\hat{p}_i}$$

Over n years an event with annual probability p is exceeded about np times, so a value exceeded i times out of n estimates $\hat{p} \approx i/n$. Dividing by $n + 1$ instead gives the Weibull plotting position.