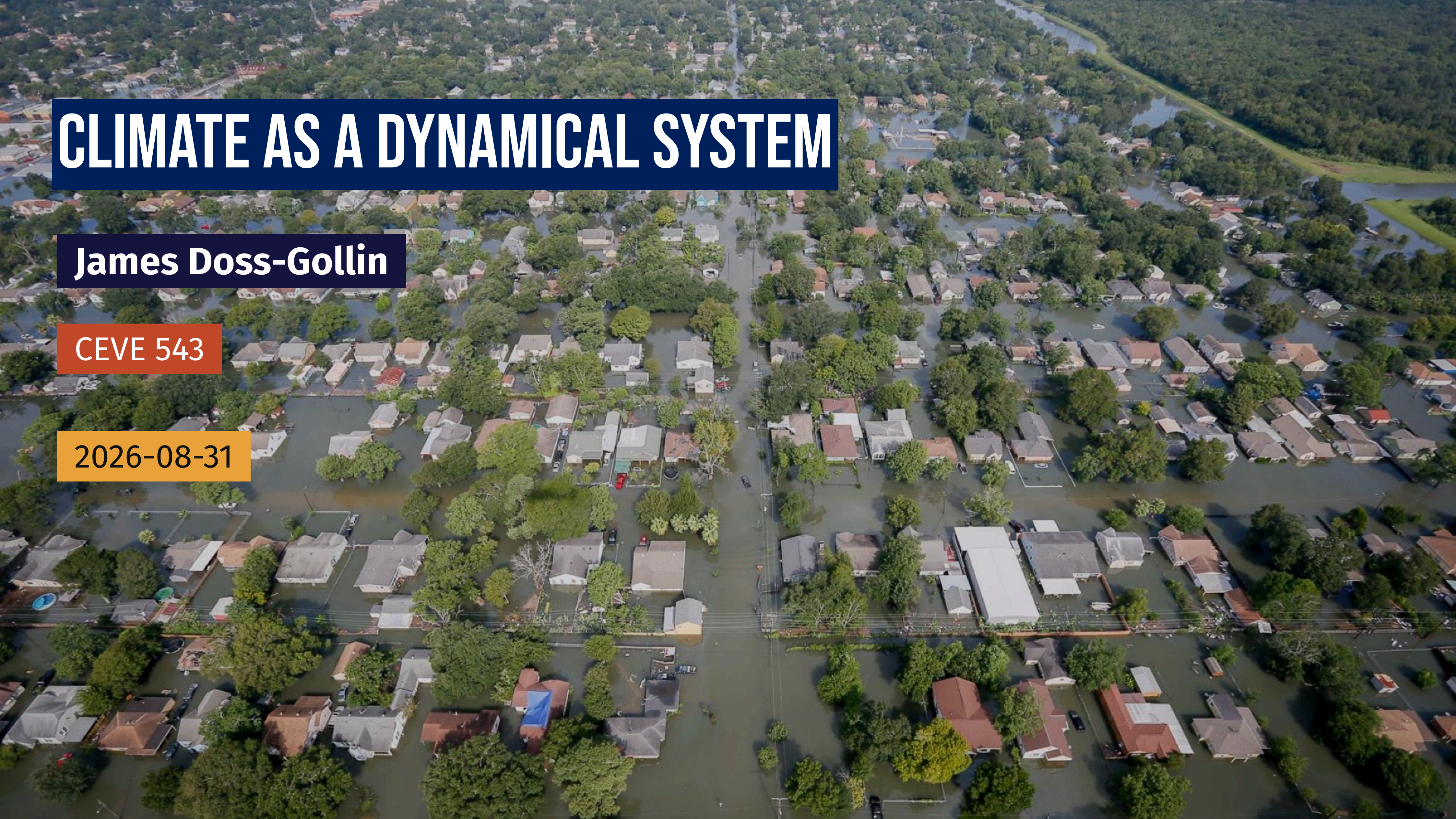

CLIMATE AS A DYNAMICAL SYSTEM

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CEVE 543

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WHY SIMPLE MODELS?

Deterministic dynamical systems, with and without random forcing, can produce much of the behavior we saw in the first deck: persistence, clustering, apparent trends, regime-like shifts.

These models help you build intuition about what kinds of behavior are possible and how to describe them mathematically.

A DYNAMICAL SYSTEM

A dynamical system has a **state** $X = (X_1, \ldots, X_p)$ and a **rule** F that determines how the state changes. In continuous time:

$$\dot{X}_i = F_i(X_1, \ldots, X_p), \qquad 1 \leq i \leq p$$

In discrete time:

$$X_{t+1} = G(X_t)$$

The system is **nonlinear** when the rule involves products or powers of the state variables, like $x \cdot z$ or $x(1 - x)$. Linear systems cannot produce chaos.

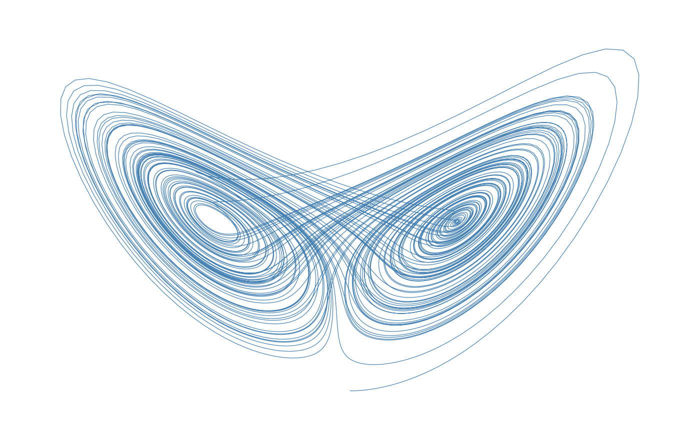
IMPORTANT TERMS

- **Trajectory:** the sequence of states the system visits over time.
- **Phase space:** the space of all possible states.
- **Attractor:** the set of states the system settles onto over long times.
- **Sensitive dependence:** small changes in initial conditions lead to diverging trajectories.

LORENZ AND SENSITIVE DEPENDENCE

In 1961, Edward Lorenz restarted a weather simulation from a printout that rounded the state to three decimal places instead of the six stored in memory. The restarted run diverged completely from the original within a few simulated days.

For supplementary notes on this discovery, read James Gleick's *Chaos* (1987), chapters 1-2.



$$\dot{x} = 10(y - x)$$

$$\dot{y} = x(28 - z) - y$$

$$\dot{z} = xy - \frac{8}{3}z$$

THE LOGISTIC MAP

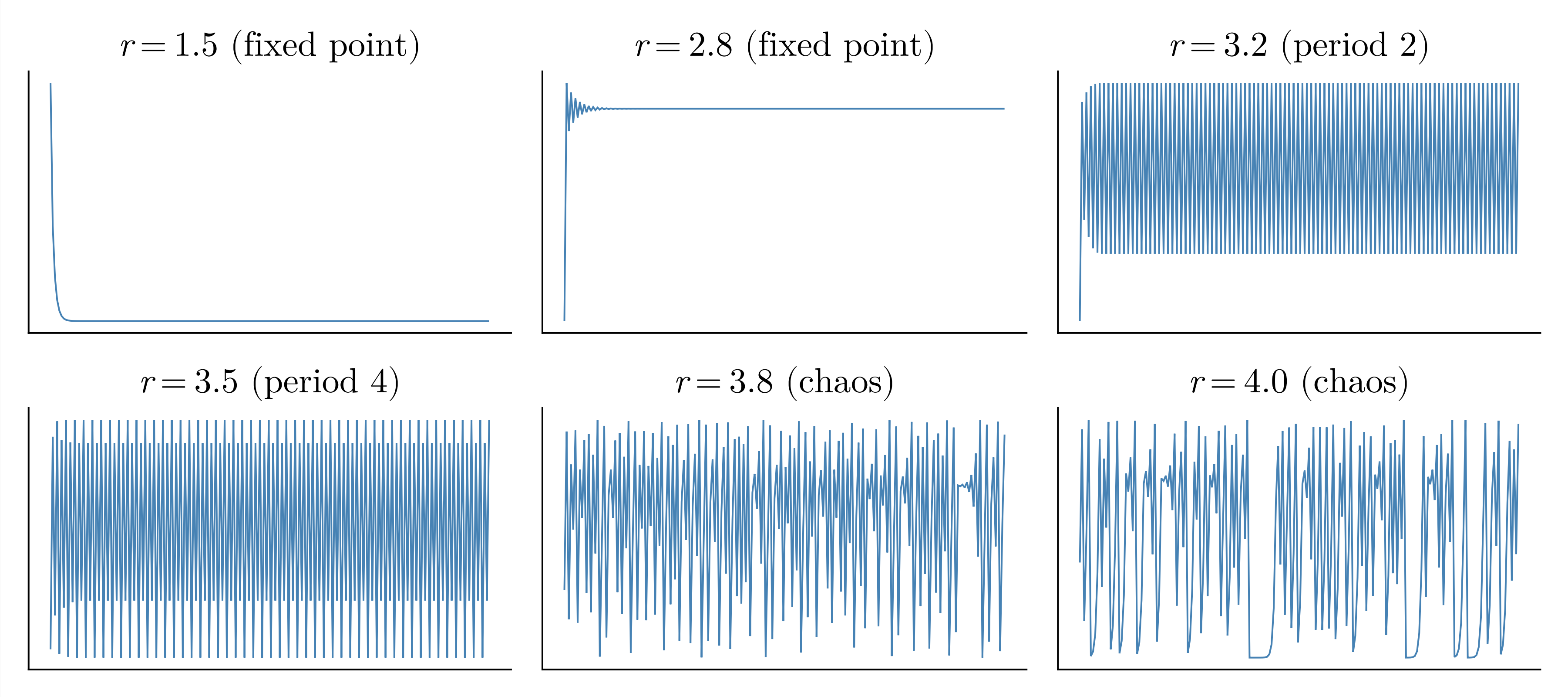
A model of bounded growth:

$$x_{t+1} = r\, x_t\, (1 - x_t)$$

The parameter r controls how strongly the system overshoots.

```
function logistic(r; n=200, x0=0.4)
    x = zeros(n)
    x[1] = x0
    for t in 1:(n - 1)
        x[t + 1] = r * x[t] * (1 - x[t])
    end
    return x
end
```

SIX VALUES OF r, SIX BEHAVIORS



WHAT DO WE SEE?

A **fixed point**: the system settles to a constant.

A **periodic orbit**: the system repeats exactly every k steps.

Chaos: the system is deterministic but sensitive to initial conditions, so small differences grow exponentially and the trajectory never repeats.

MEASURING MEMORY

The **autocorrelation** at lag h measures how much today's value tells you about the value h steps later (Kantz, 2004, §2.2):

$$\rho_h = \frac{\mathrm{Cov}(X_t, X_{t+h})}{\mathrm{Var}(X_t)}$$

If $\rho_h = 0$, the process has no memory at lag h. If $\rho_h > 0$, high values tend to follow high values.

WHAT ABOUT RANDOMNESS?

Processes no model can predict force real systems: weather, wind bursts, volcanic eruptions.

Adding a noise term ε_t to a deterministic rule gives a **stochastic dynamical system**.

DISCUSSION

Given a single observed record of finite length: why is it hard to reconstruct the system that produced it?

LOGISTICS

- Lab 1 due Friday
- Set up the software for Lab 2 before Friday’s session
- I posted this week’s practice problems